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Could Do Better: The Effectiveness of Incentives and Competition in Schools*

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Abstract

This paper studies the effects of incentive mechanisms and of the competitive environment on the interaction between schools and students, in a set-up where their effort affects the students' educational attainment. We show that increasing the power of the incentive scheme and the effectiveness of competition may have the counterintuitive effect of lowering the students' attainment. In a simple dynamic set-up, where the reputation of the schools affects recruitment, we show that increased competition leads to segregation of pupils by ability.

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1 Introduction

Policy makers across the world are trying to improve the performance of the education system. Reforms have centred around ideas which the economics literature has identified as essential in influencing the performance of commercial organisations, such as the provision of incentives linking individuals' monetary reward to their performance and the creation of vestigial forms of competition between institutions.

There are however fundamental differences between commercial organisations and educational institutions.¹ The lack of a monetary measure for the performance of an education institution is an obvious one, but, no less importantly, education establishments (schools and universities) use a customer-input technology (Rothschild and White 1993, 1995): the characteristics of the users of an educational institution, that is of its customers, affect the quality of its output. This is true both of the pupils' ability,² and of the effort they exert while at school.³

The aim of the paper is to illustrate the consequences of these features of the education production process on the way incentive schemes and competitive mechanisms operate. We study the interaction between schools, students, and employers. Employers form expectations on the ability of the school leavers, and offer them a wage which depends on this expectation. Schools, and their teachers, make investments; these influence the schools' results, both directly and indirectly, by attracting abler students. Last but not least, the students themselves exert effort while at school, thus affecting

¹An exhaustive discussion of the distinctive economic features of the education industry is in Winston (1999).

²This is known as the "peer group" effect: students learn better if they are in a group of abler students. This is a reasonably well documented phenomenon, see Moreland and Levine (1992) for a survey from a psychology/education viewpoint, and Summers and Wolfe (1977), Henderson et al. (1978) for early economic empirical studies and Epple et al. (2000) and Zimmer and Toma (2000) for more recent ones. From a theoretical viewpoint, the analyses of Arnott and Rowse (1987) and de Bartolomé (1990) were among the first to take the peer group effect explicitly into account.

³Empirical studies rarely include students' effort as an independent input in the education production function: this is probably due to the difficulty of obtaining independent measures of effort. The one exception we are aware of is Bønnesronning's (1998) study of Norwegian schools. His analysis provides positive and significant estimates of student effort on student achievement. Also he finds that students effort is affected both by school factors and family characteristics. At a theoretical level, the literature is equally scarce; the one analysis we are aware of is the undeservingly little noticed paper by Correa and Gruver (1987).

their own qualification. As the paper shows, the interaction among these groups of agents is very complex, and it is shaped powerfully, and in often-unexpected ways, by the environment created by the incentive mechanisms and the competitive framework in which schools operate. Both of which are of course shaped by government policy.

Students maximise their expected future earnings, reduced by the cost of effort, and employers maximise expected profits. If these objectives are canonical within economic theory, there is no standard choice for the objective function of schools. The assumption of this paper is that a school aims at maximising the average qualification of its students, reduced by the investment cost.⁴ As any teacher knows, this is realistic and plausible, even in the absence of any explicit mechanism linking pay to performance. However, government policy can strengthen the importance of a school's results in the school's objective function, for example by explicitly linking the teachers' remuneration and chances of promotions with the results obtained by the pupils at their school.⁵

In Section 3, we study the benchmark case of an isolated school. We show that the theoretical link between the provision of incentives and the educational output is ambiguous: more powerful incentives may have the perverse effect of lowering the effort exerted by the students .

In Section 4, we study two competing schools. Introducing competition, typically by freeing parents from the rigid link between their residence and the school attended by their children, is a major plank of many reform proposals, including those referred to in footnote 5. While there are some theoretical analyses of the role of competition between state and private

⁴Size may of course also matter. We prefer to concentrate on students' results and therefore we fix exogenously the size of the schools we consider.

⁵ There are many examples. In several US states and districts teachers are offered bonuses and/or salary increases for meeting or exceeding academic objectives (as an example, a pilot programme in Denver, Colorado, involving 15 schools and 450 teachers who could receive up to \$1,500 in bonuses, based on (i) increases in student performance on standardised tests, (ii) increases in student performance on teacher-developed assessments, and (iii) increases in teachers' skills and knowledge; see BRT and NAB 2001). This principle has been enshrined in the US blueprint for school reform (US Congress 2002). In the UK, the Labour government is implementing a performance related pay system for teachers and head teachers (see www.dfes.gov.uk/teachers). In the Australian state of Victoria, union and the state government agreed in 2001 to link teachers' promotions to improvements in student learning monitoring via state-wide testing (Victorian Government 2001). Interesting policy experiments were conducted in Israel recently which aimed to measure the responsiveness of students results to financial incentives for groups (Lavy 2002a) and for individual teachers (Lavy 2002b).

schools (for example Epple and Romano’s (1998) analysis of the effects of a vouchers scheme), and empirical analyses of the effects of competition both between state and private institutions and for institution within the public sector,⁶ theoretical analyses of competition *within* the public sector are rare.⁷

In Section 4.1, we assume that schools try to attract high ability pupils. They can do so both by improving their teaching environment and by undertaking activities that do not improve the quality of their teaching; examples are “marketing” expenditures, mail shots, fairs, brochures, open days, and so on, and the administration of interviews and admission tests. And, as our examples show, it may well happen that an increase in the power of the incentives scheme has the perverse effect of a *reduction* in the average qualification obtained by the students.

The effectiveness of the competitive mechanism may also have ambiguous effects on the schools’ result. When the students become more responsive to changes in the schools, schools devote more effort to recruiting them, increasing the marginal cost of teaching effort, thus reducing the latter and the students’ attainment.

An important factor influencing parents’ preference for schools is the reputation created by the past performance.⁸ The simple dynamic model based on this idea in Section 4.2 shows that reputation can be self-perpetuating: abler children attend the school which performed better in the past, and because this school has abler pupils, it will also perform better in the future, and so on. However, the effects of reputation on the student-teacher interaction are ambiguous. It may happen that the “better” school (namely the school with superior results) is in fact the school where students and teachers work less hard: results are better simply because abler students are enrolled. Competition also creates segregation by ability: the gaps in average ability between the two schools and in their result increase as parental

⁶For example, Borland and Howsen (1992), Hoxby (1994), and Dee (1998), show that additional competition from private schools improve outcomes for students in public schools in the US. Hoxby (2000) shows that schools choice in the US raises school productivity. Bradley et al. (2001) show that, over the period 1993-1998, competition among secondary state schools in the UK led to increases in efficiency.

⁷De Fraja and Iossa (2002) study the case of competition between two not-for-profit universities located in different towns.

⁸In the UK, the government has published all the state schools results in a variety of dimensions. Several newspapers use this data to construct highly popular –and influential– “League Tables” of the various schools. Gibbons and Machin (2002) have documented the effects of the rankings obtained from these tables on house prices in England and Wales.

choice becomes more responsive to past results.

One interpretation of our results illustrating ambiguous effects of incentive schemes is that they may provide an explanation for empirical studies which suggest that additional investment in schools resources may not have an impact of results.⁹ To the extent that more powerful incentives are costly – which is the case if teachers are risk averse –, then, according to our analysis, an increase in the power of incentive schemes increases the resources available to schools, while having ambiguous effects on performance. In a naive view of the world, *ceteris paribus*, additional resources would improve results; but, if the additional resources also affect the trade-offs of the agents participating in the education process, their actions will also change, and therefore the assumption of *ceteris paribus* cannot be warranted, and the full analysis must also include the potentially offsetting effects operating indirectly via the actions of teachers and students.

The paper is organised as follows: in Section 2, we present the actors of the model: students, schools, and employers. Section 3 studies the benchmark case of a monopoly school. In Section 4.1 we study the interaction between two schools, which can use resources to attract students. Finally in Section 4.2, we consider a simple model of dynamic interaction between schools, where demand for places responds to past results. Concluding remarks are in Section 5.

2 The model

We study the education system of an economy. This comprises three groups of agents: students, schools and employers. They are described in detail in the subsections below.

2.1 Students

There is a continuum of individuals in the economy, identical in every respect except their ability. This is measured by a unidimensional parameter $\theta \in \Theta \subseteq \mathbb{R}_+$, distributed according to a differentiable function $\Phi(\theta)$,

⁹Hanushek (1986) is an influential early survey. More recently, similar result are obtained by Betts (1995), Heckman et al (1996), and Dearden et al. (1997). Some of these studies are analysed by Card and Krueger, who conclude that “there is some evidence that school resources affect earnings and educational attainment, although much uncertainty remains in the literature” (1998, pp. 39).

with $\Phi'(\theta) = \phi(\theta)$. The average ability of the individuals is given by $\bar{\theta}$, $\bar{\theta} = \int_{\theta \in \Theta} \theta \phi(\theta) d\theta$.

All individuals attend school, and, subsequently, enter the labour market. When at school, they exert effort $e \in E \subseteq \mathbb{R}_+$. This measures how diligent she is, how hard she works and so on; it also includes parental effort, such as checking homework. e has a utility cost measured by a function $\psi(e)$, increasing and convex, $\psi'(e), \psi''(e) > 0$. We assume that, while at school, and, therefore, when they choose their effort, students know the ability distribution of the students' enrolled at their school, but not their own ability. This tallies with the idea that the education process is one of the ways in which we learn our ability. The advantage of this specification is that all students in a given school exert the same level of effort (which simplifies the analysis and would not necessarily happen if students knew their own ability).¹⁰

A student leaves school with a qualification, described in detail in Section 2.2, and enters the labour market. Here, she receives a wage, which depends on the employers' expectation of her productivity, which in turn depends on her qualification and the employers' inference about her ability. This is derived below, in Section 2.4. A student's objective function is the maximisation of the difference between expected future wage and current effort.

2.2 Schools

Students attend school and leave with a qualification. This is a variable q taking values in a continuum:¹¹ $q \in Q \subseteq \mathbb{R}_+$. The realised value of q is

¹⁰A more realistic assumption could entail dividing the entire ability set Θ in subsets, (for example, low, medium, and high ability), and positing that students know which subset they are in, but not the exact value of their ability. This would imply that all students in the same interval exert the same effort level, and would add additional complexity with no added insight. At the opposite extreme, we could have assumed that students only know the ability distribution in the economy. This would make no difference: the standard requirement of subgame perfection implies that, in equilibrium, agents understand the game played by the schools and can therefore *deduct* the ability distribution in each school, even though they cannot observe it directly.

¹¹It should be noted that, in some cases, qualification is a discrete variable (for example, in the UK, A/B/C/D/E/F at school and I/IIi/IIii/III/Ordinary at university). Even when the qualification is discrete, it is often the case that institutions make it finer in unofficial ways (such as providing a transcript of the examination marks, writing reference letters which specify the "quality" of a student's degree, giving the rank in her cohort, and so on).

affected by four factors.

The first two may differ from individual to individual: her effort while at school, e , and her ability, θ . The other two characterise the school and take the same value for all the students at a given school i : the quality of the teaching $s_i \in S \subseteq \mathbb{R}_+$, and the average ability of the students in the school, $\bar{\theta}_i$. The latter captures the “peer group effect”. The variable s_i , on the other hand, captures the idea that a school can make investments which affects its quality. The nature of the school’s investments is quite complex. We posit that they can be of three conceptually distinct types, depending on their timing and on their effects. Specifically, part of a schools’ investment is constituted by activities which are fixed before the students are enrolled at school and have therefore the feature of a durable investment. These can themselves be divided into two parts; some of it improves the quality of the school’s teaching: the quality of buildings, classroom equipment, computers, sporting facilities, teachers’ qualifications and so on. A second part does not directly affect the qualification of its students: in this category belong expenditures which are aimed towards student recruitment, such as advertisement brochures, the organisation of visits for prospective pupils and their parents, and generally what can be classified as “marketing” in a wider sense. The third component of the school’s effort is instead expended once the students are at school, and it affects directly the process of learning. It is given by the teachers’ effort in the activities in the classroom, by the time they spend outside teaching hours to prepare lessons, to assess the students’ work, to meet parents, and so on. By its very nature, while the school’s investments are observable by parents, they are not contractible, and so neither can the school commit itself to a specific level for them prior to enrolling the students, nor can a government agency, or indeed the parents, require the school to make them, or agree to reward the school for undertaking them.

We capture formally the discussion in the above paragraph by letting the variable $s \in S \subseteq \mathbb{R}_+$ measure the school’s teaching quality and we assume it to be given by:

$$s = \mu z + x, \quad 0 < \mu < 1, \quad (1)$$

where z is the effort exerted *before* students join the school, and x the effort exerted *while* students are in the school. The assumption that $\mu < 1$ implies that a fraction $(1 - \mu)$ of the durable investment is directed towards activities which do not directly affect the quality of its teaching, and so do not affect directly students attainment.

Thus we can let a student's qualification be denoted by $q(e, \theta; s_i, \bar{\theta}_i)$ (an error term could be added without altering the result), with all partial first derivatives are positive: *ceteris paribus*, a student obtains a better qualification who works harder, who is abler, who receives better teaching, and who has abler classmates. We simplify the analysis by taking the following specific functional form for q :

$$q(e, \theta; s_i, \bar{\theta}_i) = (e + \theta) g(s_i, \bar{\theta}_i), \quad (2)$$

where $g(s, \bar{\theta}) > 0$ satisfies $g_s(\cdot), g_{\theta}(\cdot) > 0$ and $g_{ss}(\cdot), g_{\theta\theta}(\cdot) < 0$. This formulation has a very natural interpretation: a student's qualification is proportional to her individual characteristics, effort and ability (note that taking their sum amounts to little more than a normalisation). The coefficient of proportionality is given by the school's characteristics: teaching quality s_i and average ability of the pupils, $\bar{\theta}_i$. In view of this, it is natural to refer to $g(\cdot)$ as a measure of the school's quality.

A school pursues an objective function which depend positively on the average qualification of its students and negatively on the teaching effort:

$$\lambda \int_{\theta \in \Theta} (e(\theta) + \theta) g(\mu z + x, \bar{\theta}) \phi(\theta) d\theta - \zeta(x + z), \quad (3)$$

where $e(\theta)$ is the effort level exerted by students of ability θ . The function $\zeta(x + z)$ is increasing and convex, $\zeta'(\cdot), \zeta''(\cdot) > 0$, implying increasing marginal disutility of effort. Additivity between the two components of effort and between the disutility of effort and the students' average qualification are simplifying assumptions. In view of the functional form (2), the objective function of the school (3) can be written as:

$$\lambda (\bar{e} + \bar{\theta}) g(\mu z + x, \bar{\theta}) - \zeta(x + z), \quad (4)$$

where \bar{e} is the average effort exerted by the school's students. λ is an important parameter in the paper. It measures the importance of the students' qualification for the school's payoff relative to the cost of effort. It is affected by the policy instruments used by the government to provide incentives to teachers, and we therefore couch our analysis in terms of the effects of changes in λ on the behaviour of the education system.¹²

¹²Clearly, λ could differ across schools. This creates a further source of interaction, as it might, for example, affect the types of teachers appointed by schools with different values of λ . This is a possible topic for further research.

Schools choose their investment and their effort. In the benchmark case, studied in Section 3, the school takes as given the characteristics of the students it enrolls. Subsequently, in Section 4, we allow schools to affect the ability of its pupils.

2.3 The game

To sum up the description of the model, we study the following three stage game:

- in the first stage, schools choose their investment, z .
- in the second stage, students and schools simultaneously and independently choose their effort levels, e and x ;
- in the third stage, described and solved in the next subsection, employers observe each school leaver's qualification and make her a wage offer.

We study the pure strategy perfect Bayesian equilibria of this game, which in our case, implies that in each stage players correctly anticipate the actions which will be chosen in the subsequent stages, and that employers' beliefs about the students' ability are consistent with the strategies employed by the students and schools in the first and second stage.

2.4 Employers

The focus of our paper being on the interaction between schools and students, we model the labour market as simply as possible. We assume that an individual's output in the labour market depends on the qualification obtained at school and on her innate ability:

$$\pi(q, \theta). \tag{5}$$

This relationship is deterministic; adding an error term would not alter the qualitative nature of the analysis. Moreover, there are no externalities or economies of scale in production, so that a worker's output does not depend on the characteristics or the number of her fellow employees. There is a competitive labour market, so that employers bid each worker's wage up until they make zero expected profits from employing that worker.

This implies that an individual's wage is given by the expected value of that individual's output:

$$w = \int_{\theta \in \Theta} \pi(q, \theta) f(\theta; q) d\theta.$$

where $f(\theta; q) \in \mathbf{R}_+$ is the density of the representative employer's belief about the ability of a student whose qualification is q , the information available at the time that worker is hired.

We consider pure strategies only, which seems natural in the present set-up. Therefore, all students in a given school exert the same level of effort, say \bar{e} , and therefore employers can infer exactly a student's ability from her qualification; this is so because, given the school's quality, there is a one-to-one relationship between ability and qualification. It follows that $f(\theta; q)$ is a discrete distribution, with the entire mass on a single point $\theta \in \Theta$. The requirement that the equilibrium is Bayesian implies that the employers' beliefs about the students' ability are consistent with the strategies employed by the students and schools in the earlier stages, and therefore the mass point of the employers' belief is the true value of a student's ability. To sum up, for an individual with ability θ and qualification q from school i :

$$w = \pi(q, \theta) = \pi((\bar{e} + \theta)g(\mu z_i + x_i, \bar{\theta}_i), \theta).$$

3 A benchmark case: the “monopoly school”.

In this section we study the monopoly school. This is not only a realistic benchmark, applying as it does to all situations where the number and characteristics of the students attending a school are exogenously given, but it also constitutes the foundation for the study of the more general case where schools interact with each other. Although the interaction between a school and its students is a game with an infinite number of players, it can be studied in fairly simple terms, in view of the fact that students do not know their ability and therefore behave in the same way.

Let z be fixed and known to all concerned. Consider the representative student first. She takes as given the effort choice of the school and of all her fellow students, as well as correctly anticipating how employers will behave when offering wages, and she maximises her expected wage, net of the utility cost of her effort. Her maximisation problem is stated formally in the next result.

Lemma 1 *Let \bar{e} be the average effort exerted by the other students. The maximisation problem of a student at school i is given by:*

$$\max_{e \in E} U(e) \equiv \int_{\theta \in \Theta} \pi((e + \theta)g(\mu z + x, \bar{\theta}), e + \theta - \bar{e}) \phi(\theta) d\theta - \psi(e). \quad (6)$$

Proof. The maximum wage that an employer is willing to pay to a student who has qualification q is given by $\pi(q, \hat{\theta})$. q satisfies

$$q = (e + \theta)g(\mu z + x, \bar{\theta}), \quad (7)$$

and $\hat{\theta}$ is the employer's inference about the student's ability, and is therefore given by:

$$\hat{\theta} = \frac{q}{g(\mu z + x, \bar{\theta})} - \bar{e}. \quad (8)$$

Substitute (7) into (8) to obtain that an employer's inference about a student with qualification q is given by $e + \theta - \bar{e}$. The rest of the Lemma follows. ■

The lemma captures the fact that a student tries to manipulate the signal determined by her qualification, by working, as it were, harder than her colleagues. Of course, in equilibrium, every student is trying to do precisely this, and therefore all students exert exactly the same level of effort, so that no student is in fact able to manipulate the signal provided by her qualification.

The school's decision of effort is more straightforward: a school takes the effort level of the students as given, and maximises (4). The next result characterises the Nash equilibrium of this stage of the game.

Proposition 1 *The Nash equilibrium of the second stage game¹³ is given by the simultaneous solution of the following two equations in \bar{e} and x :*

$$\int_{\theta \in \Theta} [g(\cdot) \pi_q((\bar{e} + \theta)g(\cdot), \theta) + \pi_\theta((\bar{e} + \theta)g(\cdot), \theta)] \phi(\theta) d\theta = \psi'(\bar{e}), \quad (9)$$

$$\lambda(\bar{e} + \bar{\theta})g_s(\mu z + x, \bar{\theta}) = \zeta'(x + z). \quad (10)$$

Proof. Derive the best reply function of the representative student first. Differentiation of the LHS in (6) with respect to \bar{e} yields the first order condition:

$$U'(e) = \int_{\theta \in \Theta} [g(\cdot) \pi_q((e + \theta)g(\cdot), e + \theta - \bar{e}) + \pi_\theta((e + \theta)g(\cdot), e + \theta - \bar{e})] \phi(\theta) d\theta - \psi'(e). \quad (11)$$

¹³We assume that the relevant second order conditions are satisfied. For this to be the case, a sufficient assumption is

$$\pi_{\theta q}(q, \theta) \leq -\frac{1}{2} \left[g(s, \bar{\theta}) \pi_{qq}(q, \theta) + \frac{\pi_{\theta\theta}(q, \theta)}{g(s, \bar{\theta})} \right],$$

for every $q \in Q$, for every $\theta \in \Theta$, for every $s \in S$.

In equilibrium, $e = \bar{e}$, and (11) becomes (9). Consider the school next: to obtain (10) differentiate (4) with respect to x , noting that the second order condition is satisfied, since $g_{ss}(\cdot) < 0$ and $\zeta''(x) > 0$. ■

The equilibrium derived in Proposition 1 can be studied with a graphical analysis, without the consideration of explicit functional forms.

We begin by determining the effect of a change in the school's characteristics on the effort exerted by the representative student.

Lemma 2 *If $\psi''(\bar{e}) > \int_{\theta \in \Theta} g(\cdot) [\pi_{qq}(\cdot) g(\cdot) + \pi_{\theta q}(\cdot)] \phi(\theta) d\theta$, then $e = \bar{e}$ implies $\frac{de}{d\bar{e}} < 1$.*

Proof. Total differentiation of (11) yields:

$$\frac{de}{d\bar{e}} = \frac{\int_{\theta \in \Theta} [\pi_{q\theta}(\cdot) g(\cdot) + \pi_{\theta\theta}(\cdot)] \phi(\theta) d\theta}{U''(e)},$$

where

$$U''(e) = \int_{\theta \in \Theta} \{g(\cdot) [\pi_{qq}(\cdot) g(\cdot) + 2\pi_{\theta q}(\cdot)] + \pi_{\theta\theta}(\cdot)\} \phi(\theta) d\theta - \psi''(e). \quad (12)$$

For this to be less than 1, it must be:

$$\int_{\theta \in \Theta} [\pi_{q\theta}(\cdot) g(\cdot) + \pi_{\theta\theta}(\cdot)] \phi(\theta) d\theta > \int_{\theta \in \Theta} \{g(\cdot) [\pi_{qq}(\cdot) g(\cdot) + 2\pi_{\theta q}(\cdot)] + \pi_{\theta\theta}(\cdot)\} \phi(\theta) d\theta - \psi''(e).$$

which establishes the Lemma. ■

Lemma 2 allows us to determine diagrammatically the equilibrium value of \bar{e} in a Cartesian diagram with \bar{e} and e on the axes. Note that $e(\bar{e})$ can be interpreted as the best reply function of an individual student: given that the rest of the students exert effort level \bar{e} , $e(\bar{e})$ is that student's optimal response. At the intersection with the 45 degree line, where $e = \bar{e}$, the student exerts the average effort level, and so every student will also do so. When $\frac{de}{d\bar{e}} < 1$ the solid curve in Figure 1 intersects the 45 degree line from below, as depicted. The condition required for the term $\frac{de}{d\bar{e}}$ to be less than 1 is weak: essentially it is satisfied as long as the second cross derivative $\pi_{q\theta}(\cdot)$ is not "too high", that is, if the effect of qualification on productivity does not raise "too much" with ability. In what follows, we assume it to be satisfied. The diagram can be used to illustrate the effect of a change in the school's characteristics, $g(\cdot)$ on the effort exerted by the representative student. Total differentiation of (11) gives:

$$\frac{de}{dg(\cdot)} = \frac{\int_{\theta \in \Theta} [\pi_q(\cdot) + [\pi_{qq}(\cdot) g(\cdot) + \pi_{\theta q}(\cdot)] (e + \theta)] \phi(\theta) d\theta}{-U'''(e)}. \quad (13)$$

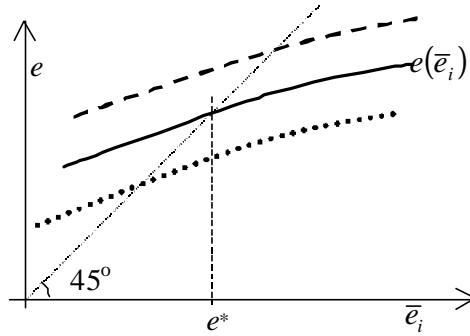


Figure 1: The equilibrium of the game among students

The above can have either sign: if it is positive (negative), then the curve $e(\bar{e})$ moves up (down) as a consequence of an increase in g , as depicted by the dashed (dotted) locus in Figure 1.

When (13) is positive, school quality and students' effort are *complements*. There is a kind of “multiplier” effect of an increase of a school's quality: a better school increases the marginal benefit of a student's effort, making it worthwhile for her to work harder, in order to improve her signal to the market: but, if one student works harder, then all students do. This improvement in students' effort enhances the improvement in qualification due to the increase in the schools' teaching quality. If the sign of (13) is negative, school quality and students' effort are *substitutes*: an increase in g brings about downward shift of the curve $e(\bar{e})$ and therefore a reduction in students' effort. Students respond to an increase in the school quality by reducing their own effort, which (partially) offsets the beneficial effect of increased quality.

This discussion, of course, only illustrates part of the story, because the school itself responds to changes in the students' behaviour. To study this, consider the game between a school and its students; we derive in Lemma 3 the slope of the best reply functions of the school and the students,¹⁴ in the (x, \bar{e}) cartesian plane. We also determine the comparative statics effect of changes in the power of the incentives, which, as discussed above, we proxy with a change in λ , and of changes in z , the investment level determined in

¹⁴Note the slight abuse of terminology: students do not take actions in concert, and the term “the students' best reply function” is to be interpreted as “the equilibrium level of effort of the students as determined in the subgame where students choose effort taking the school quality as given”.

the first period.

Lemma 3 *The students' best reply function satisfies:*

$$\frac{d\bar{e}}{dx} = \frac{d\bar{e}}{de} \frac{de}{dg(\cdot)} g_s(\cdot), \quad (14)$$

$$\frac{d\bar{e}}{dz} = \mu \frac{d\bar{e}}{dx}, \quad (15)$$

$$\frac{d\bar{e}}{d\lambda} = 0. \quad (16)$$

The school's best reply function satisfies:

$$\frac{dx}{d\bar{e}} = \frac{\lambda g_s(x, \bar{\theta})}{-(\lambda(\bar{e} + \bar{\theta}) g_{ss}(\cdot) - \zeta''(x))} > 0, \quad (17)$$

$$\frac{dx}{dz} = -\frac{\mu \lambda (\bar{e} + \bar{\theta}) g_{ss}(\cdot) - \zeta''(\cdot)}{\lambda(\bar{e} + \bar{\theta}) g_{ss}(\cdot) - \zeta''(\cdot)} < 0, \quad (18)$$

$$\frac{dx}{d\lambda} = \frac{(\bar{e} + \bar{\theta}) g_s(\cdot)}{-(\lambda(\bar{e} + \bar{\theta}) g_{ss}(\cdot) - \zeta''(x))} > 0. \quad (19)$$

Proof. (14), (15) and (16) are immediate from total differentiation of (11), having substituted $e = \bar{e}$.

With regard to the school best reply function, totally differentiating the first order condition (10) with respect to x, \bar{e}, λ and z yields:

$$\begin{aligned} & (\lambda(\bar{e} + \bar{\theta}) g_{ss}(\cdot) - \zeta''(\cdot)) dx + \lambda g_s(\cdot) d\bar{e} + \\ & (\bar{e} + \bar{\theta}) g_s(\cdot) d\lambda + (\lambda \mu (\bar{e} + \bar{\theta}) g_{ss}(\cdot) - \zeta''(\cdot)) dz = 0. \end{aligned} \quad (20)$$

And the rest of the Proposition follows easily. ■

In Figure 1, we have depicted the students' and the school's best reply functions (as the solid lines). The latter (for which we give two possible positions) is increasing, by (17); the former has slope given the sign of (14). This is the same as the sign of $\frac{de}{dg(\cdot)}$, the response of an individual student to changes in the school's characteristics, which is given in (13). It is decreasing if school's quality and student's effort are complements, increasing if they are substitutes. We have drawn Figure 2 in the heuristically plausible case in which they are complements for low values of x (and hence $g(\cdot)$) and substitutes for higher values of x and $g(\cdot)$: note that $g(\cdot)$ is the coefficient of the negative term $\pi_{qq}(\cdot)$ in the numerator of (13); clearly other shapes

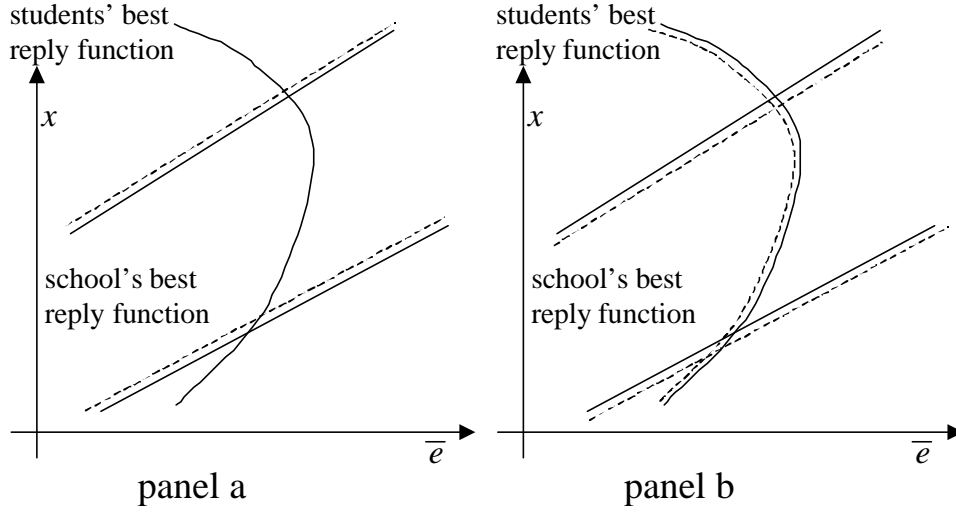


Figure 2: Effects of changes in λ (panel a) and z (panel b) on the Nash equilibrium.

are possible.¹⁵ Panel (a) illustrates the effects of an increase in λ , the power of the incentive schemes. From (16), we see that the students' best reply function does not move, and, from (19), the school responds to an increase in λ with an increase in x : the best reply curve shifts up.

The effect of a change in λ on the equilibrium depends on whether the students' best reply function is upward or downward sloping. If the school's quality and the students' effort are complements then, as shown in the lower part of panel (a) in Figure 2, both the school's and the students' effort increase, and so, clearly, will the average qualification of the students: strengthening incentives makes teachers and students work harder and improves results. However, if the school's and the students' effort are substitutes, as is the case for the higher curves in the diagram, then the best reply function of the students is sloping backwards, and the students' effort decreases in response to an increase in λ , and the overall effect on attainment is ambiguous.¹⁶ This is consistent with the Lavy's empirical

¹⁵The figure does not print the iso-utility curves, so as to avoid cluttering the diagrams. It is however immediate to verify that both the school and the students would benefit if they could find a way to commit to higher levels of effort, as shown by Correa and Gruver (1987), irrespective of whether the school's and the students' efforts are substitutes or complements.

¹⁶It could happen that, at their intersection, the school's best reply function is steeper than the students' best reply function. In this case, an increase in λ would unambiguously

findings (2002): he studies a policy experiment in Israeli schools, and finds that teachers improve their effort in response to financial incentives, and that students' attainment improves as a result.

In panel (b) we consider effects of changes in z , which is determined in the first stage. From (18), an increase in z shifts the school's best reply function down, and the students' best reply function to the left. The school's reduces the component x of its effort; the students also reduce their effort if the school's and the students' efforts are substitutes or if, loosely speaking, the students' best reply function shifts less than the school's.

To complete the analysis of the game described at the beginning of Section 2.3, we need the determination of the variable z . This is conceptually simple, as there are no strategic actions by the students: the school simply chooses the value of z which maximises $\lambda (\bar{e}(z) + \bar{\theta}) g(\mu z + x(z), \bar{\theta}) - \zeta(x(z) + z)$, where the second stage variables, \bar{e} and x , are themselves functions of z .

4 Competition between Schools

4.1 Investment choice by schools.

To the extent that their payoff depends on the average qualification and that the latter depends on the ability of its students, schools may rationally try to improve their average qualification by recruiting abler students, if allowed to do so.¹⁷ In this section we investigate this possibility. Specifically, we assume that students (or their parents) can choose which schools to apply for and that they make their choice on the basis of the observed values of the schools' characteristics; if a school is oversubscribed, it can, to some extent, select higher ability applicants. In this subsection, the relevant school characteristic is the value of the “investment” of the schools, namely the decrease both the students' and the school's effort. A brief consideration suggests however that this equilibrium would not be “stable” under plausible adjustment mechanisms. This possibility will be disregarded in what follows.

¹⁷This is precisely what happened in systems, such as New Zealand, where competition was introduced. Schools try to be authorised to have an “enrolment scheme” whereby they can establish criteria for preference in the admission of applicants. The evidence suggest that schools attempted successfully to select pupils from socially advantaged backgrounds (Fiske and Ladd 2000, pp 216-223). According to Fiske and Ladd “the system quickly flip-flopped in some [...] areas from one in which parents and children choose schools to one in which *schools choose students*” (Fiske and Ladd 2000, p 9, our emphasis).

component z of the school's effort, and in the next subsection the past results obtained by the two schools. We have therefore two policy instruments: in addition to the strength of the incentives in place for schools and teachers, the effectiveness of the competitive mechanism between schools is also clearly affected by policy decisions.

We assume that there are two schools only and that each schools' size is fixed to $\frac{1}{2}$ of the total students population each. This simplifies the analysis and captures the main aspects of competition. The general model of Section 3 is ill suited to the analysis of situations where the distribution of individual abilities within a school is endogenously determined. We therefore introduce the following assumptions:

$$\bar{\theta}_i = \eta(z_i - z_j), \quad \eta'(\cdot) > 0, \quad i, j = 1, 2, \quad j \neq i, \quad (21)$$

$$\pi(q, \theta) = q + \theta - \frac{1}{2}q^2 - \frac{\rho}{2}\theta^2 + \alpha q\theta, \quad \rho \geq 0, \quad \alpha \in (-1, 1). \quad (22)$$

Without explicitly modelling the process by which schools affect their intake, (21) posits a functional relationship between the difference in first stage investments, z_i and z_j , and the *average ability* of their intake.¹⁸ Naturally, if a school increases its investment then it increases its average ability. Note that (21) implies $\eta(0) = \bar{\theta}$: if the two schools invest the same amount in stage 1, then their efforts cancel out and each has ability equal to the population average. The quadratic formulation for π given in (22) dovetails neatly with (21), because of its convenient property that the students' best reply function depends only on the average ability and not on the entire distribution, as is evident from direct substitution of (22) into (9), where θ appears only linearly.

To sum up, in stage 1, schools simultaneously choose z_1 and z_2 , thus determining the values of the average abilities, $\bar{\theta}_1$ and $\bar{\theta}_2$: these are observed by the two schools, the students, and the employers. Note that the only

¹⁸(21) implies that, in aggregate students respond continuously to changes in the difference in the schools' characteristics. Given our assumption that students are (ex-ante) identical, one may suppose that they would all react in the same way to exogenous changes in the parameters of their payoff functions. The theoretical industrial economics literature provides possible justifications for this continuous response: for example, students may be located in different geographical areas, so that for some them going to a given school is less convenient than for others (Hotelling 1929). Alternatively, given the interpretation of z as marketing expenditures, the Butters (1977) model of advertising can be applied by saying that the probability of parents getting to know about a school increases with the school's expenditure on "marketing", z ; if parents do not hear about a school they send their children to the local school.

interaction between schools takes place in stage 1: this is natural, once the students are at schools, what happens elsewhere is irrelevant. Therefore, in each school, once z_1 and z_2 are fixed, the game analysed in Section 3 is played in isolation.

Lemma 4 *The first order conditions for the first stage problem are given by:*

$$\left[\left(\frac{\partial \bar{e}_i}{\partial \bar{\theta}_i} + 1 \right) \eta' (0) + \frac{\partial \bar{e}_i}{\partial z_i} \right] g(\cdot) + (\bar{e}_i + \bar{\theta}_i) [g_{\bar{\theta}}(\cdot) \eta' (0) - g_s(\cdot) (1 - \mu)] = 0, \\ \text{for } i, j = 1, 2.$$

Proof. In the second stage, the values of z_i are fixed and therefore in the first stage school i maximises:

$$\max_{z_i \in Z} W_i(z_i, z_j) = \lambda (\bar{e}_i(\bar{\theta}_i(z_i, z_j), z_i) + \bar{\theta}_i(z_i, z_j)) g(x_i(\bar{\theta}_i(z_i, z_j), z_i) + \mu z_i, \bar{\theta}_i(z_i, z_j)) \\ - \zeta(x_i(\bar{\theta}_i(z_i, z_j), z_i) + z_i). \quad (23)$$

Differentiating (23) which respect to z_i we get the following first-order condition:

$$\lambda \left[\left(\frac{\partial \bar{e}_i}{\partial \bar{\theta}_i} \frac{\partial \bar{\theta}_i}{\partial z_i} + \frac{\partial \bar{e}_i}{\partial z_i} + \frac{\partial \bar{\theta}_i}{\partial z_i} \right) g(\cdot) + (\bar{e}_i + \bar{\theta}_i) \left(g_s(\cdot) \left(\frac{\partial x_i}{\partial \bar{\theta}_i} \frac{\partial \bar{\theta}_i}{\partial z_i} + \frac{\partial x_i}{\partial z_i} + \mu \right) + g_{\bar{\theta}}(\cdot) \frac{\partial \bar{\theta}_i}{\partial z_i} \right) \right] \\ - \zeta'(\cdot) \left(\frac{\partial x_i}{\partial \bar{\theta}_i} \frac{\partial \bar{\theta}_i}{\partial z_i} + \frac{\partial x_i}{\partial z_i} + 1 \right) = 0. \quad (24)$$

From (21) we know that at the symmetric equilibrium, $\frac{\partial \bar{\theta}_i}{\partial z_i} = \eta'(0)$ and from Proposition 1 that the school's first order condition in the second stage is given by $\zeta'(\cdot) = \lambda(\bar{e} + \bar{\theta}) g_s(\cdot)$. Substituting both expressions into $W_i(\cdot)$ establishes the lemma. ■

The complex interaction in stage 2 described in Section 3 compounds with the stage 1 interaction between the schools, making it intractable to derive qualitative results (algebraically or geometrically). We therefore resort to numerical simulations.¹⁹ In the present set-up the use of specific, relatively simple, functional forms is a strength, not a weakness, of the modelling strategy, as we intend to illustrate the multiplicity of possible outcomes: with more general functional forms the ambiguity could only be enhanced.

¹⁹The programme and the others used in subsequent sections are available on request from the authors.

We therefore let:

$$g(s_i, \bar{\theta}_i) = ks_i + (1 - k)\bar{\theta}_i, \quad (25)$$

$$\psi(e) = \frac{\gamma}{2}e^2, \quad (26)$$

$$\zeta(x + z) = \frac{\sigma}{2}(x + z)^2, \quad (27)$$

where the parameters, ρ , k , γ , and σ are positive, and $k < 1$. In the simulations we also restrict our attention to symmetric equilibria, where the two schools choose the same value of their long term investment.²⁰ This implies that all is relevant for the determination of a symmetric equilibrium is the slope of the function η when evaluated at 0, $\eta'(0)$. A higher value of $\eta'(0)$ denotes a fuller response of the average ability to difference in the schools. This, clearly, is a variable that can be affected by government policy, insofar as the government can affect the degree of competition, for example by making it easier for parents to choose their school.

Table 1 illustrates a sample of the simulations. The parameters are given by $\mu = 0.4$, $k = 0.9$, $\eta'(0) = \rho = \sigma = 1$, $\gamma = 3$, and two values for α are considered. For ease of presentation, when a variable is decreasing it is printed on a grey column. Table 2 illustrates that when the power of the incentive scheme, λ , increases, the schools shift effort towards the first period: recruiting high ability students becomes more important, and schools exert more effort to recruit them. This, however, increases the marginal cost of the effort exerted in the second period. Consequently, the latter is reduced. What matters for the schools' quality is, however, the total effort they exert, $x + \mu z$. This may increase or decrease with an increase in the power of the incentives scheme, as the two parts of the Tables show. In the upper part (with $\alpha = -0.3$), the school's quality $x + \mu z$ increases with λ , which more than offsets the decreases in students' effort e , so that every student's qualification improves (the last column in the table gives the value for the average qualification). The opposite, however, may happen: the reduction in x brought about by the increase in z (in turn determined by the increase in the power of incentives), combines with a reduction in e , and as a consequence the average qualification is now lowered by increases in the power of the incentive scheme. Different combinations are in general

²⁰ Again, this restriction strengthens our argument, as the effect of strengthening incentives and increasing competition must operate only via effort, and not via the average ability of the pupils. We do consider asymmetric equilibria in the dynamic set-up of Section 4.2.

	l	z	x	$x+mz$	e	\bar{q}
$a=-0.3$	1.0	0.0310	0.6067	0.6191	0.2086	0.4303
	1.6	0.6119	0.4057	0.6505	0.2067	0.4491
	2.2	1.1678	0.2255	0.6927	0.2037	0.4739
	2.8	1.6854	0.0759	0.7501	0.1989	0.5068
$a=0.1$	0.8	0.0177	0.5541	0.5611	0.2941	0.4407
	1.1	0.4004	0.3845	0.5447	0.2929	0.4283
	1.4	0.7878	0.2092	0.5243	0.2913	0.4130
	1.7	1.1813	0.0257	0.4982	0.2889	0.3932

Table 1:
Effects of changes in the power of incentives on the equilibrium.

$h'(0)$	z	x	$x+mz$	e	
1.0	0.0779	0.7334	0.7646	0.5016	0.7393
1.1	0.2801	0.5392	0.6512	0.5115	0.6434
1.2	0.4523	0.3684	0.5493	0.5132	0.5516
1.3	0.5942	0.2227	0.4604	0.5085	0.4683
1.4	0.7077	0.1018	0.3849	0.4994	0.3962
1.5	0.7968	0.0036	0.3223	0.4882	0.3361

Table 2:
Effects of changes in the effectiveness of competition on the equilibrium.

possible; however, we have not found any other patterns with the functional forms we have posited.

As Table 2 shows, the effects of changes in the effectiveness of competition, measured by the parameter $\eta'(0)$, is similar to that of changes in the power of incentives: the effectiveness of the first period effort is enhanced, and so schools shift effort in that direction. Once at school, however, there is no effect of the schools' effort, as there was in the case of increases in λ , and, as shown by Table 2, x decreases, and may do so to an extent sufficient to reduce the overall school's quality ($x + \mu z$) and the average qualification of the students. As before, we have not found any set of parameter displaying a different pattern.

The exact role of each parameter in determining how the equilibrium responds to changes in the policy parameters is less interesting than the ambiguity in the effects of these parameters: indeed, the main message of this section is that the effects on the school results of providing schools with incentives and of making competition more effective relationship are potentially perverse. At the very least, our analysis casts doubts on the

belief that incentives are *per se* effective.

4.2 A simple dynamic model of school competition.

One of the main reasons why parents prefer a school over another is because its exam results are better; this suggests a simple natural dynamic extension of our model, one where today's demand for places at a school is affected by yesterday's results.²¹

Formally, we assume that time is divided into periods, and that, in each period, the average ability of the pupils enrolled at school i depends on the difference between the average qualification obtained by the students at the two schools in the previous period. This variable plays therefore the same role played by the z component of the school's investment, and, for the sake of simplicity, z is set to 0: a school's intake is only affected by past results. Formally, in period t , $t = 1, 2, \dots$, the average ability of the students who are enrolled in school i , $\bar{\theta}_{1,t}$, is given by an increasing function of the difference in the schools' average examination results:

$$\bar{\theta}_{1,t} = h(\bar{q}_{1,t-1} - \bar{q}_{2,t-1}),$$

where $\bar{q}_{i,t-1}$ is the average qualification of the students attending school i in period $t-1$. As with η , we have $h(0) = \bar{\theta}$; we also have $h(y) + h(-y) = 2\bar{\theta}$. We assume that school maximise the current period payoff.²²

Let $Q(\bar{\theta}_i)$ denote the reduced form average qualification of the students attending a school where the average ability is $\bar{\theta}_i$:

$$Q(\bar{\theta}_i) = g(x^*(\bar{\theta}_i), \bar{\theta}_i)(\bar{e}^*(\bar{\theta}_i) + \bar{\theta}_i),$$

where $\bar{e}^*(\bar{\theta}_i)$ and $x^*(\bar{\theta}_i)$ are the reduced form effort levels exerted in equilibrium by students and the school when the average ability in the school is

²¹Indeed the publications of school's league tables in the UK has been the main source of information for parents of prospective pupils, and one that has clearly affected the intake in many schools.

²²That is, decision makers in schools have a discount factor of 0. This allows us to treat each period as a separate game, and eliminates the possibility of equilibria of the repeated game based on trigger strategies. We feel justified in this assumption by the fact that the focus of this paper is on the interaction between schools, and not on the role of time preferences on the behaviour of schools. Since such preference are likely to be important in practice, because teachers may stay in a school for longer than a cohort of students, further research should take rigorously into account the possibility that the interaction between schools is best described by a repeated game.

$\bar{\theta}_i$ (given by the solution of (9) and (10)). In period t , the average ability of the students in the two schools is given by:

$$\bar{\theta}_{1,t} = h(Q(\bar{\theta}_{1,t-1}) - Q(2\bar{\theta} - \bar{\theta}_{1,t-1})), \quad (28)$$

$$\bar{\theta}_{2,t} = 2\bar{\theta} - \bar{\theta}_{1,t}. \quad (29)$$

Note that $\bar{\theta}_{1,t} = \bar{\theta} = \bar{\theta}_{2,t}$ is always a steady state solution of the dynamical system (28). Whether this solution is locally stable depends in general depend on the stability condition:

$$h'(0)(2Q'(\bar{\theta})) < 1.$$

Therefore, if $Q'(\bar{\theta}) \leq 0$, that is if an increase in the average ability brings about lower schools and/or students' effort which more than compensates for the increase in average ability, the symmetric equilibrium, $\bar{\theta}_{1,t-1} = \bar{\theta}_{2,t-1} = \bar{\theta}$, is locally stable. If, however, $Q'(\bar{\theta}) > 0$, that is if an increase in the average ability of a school's intake brings about an increase of the average qualification, then, for $Q'(\bar{\theta})$ and/or $h'(0)$ high enough, the symmetric equilibrium is not stable. In this case, if there are stable equilibria, they are asymmetric. Since we are interested in finding examples of these equilibria, we are again justified in restricting the analysis to the functional forms used in the previous sections, (22) and (25)-(27). In addition, we also assume that the function h is given by

$$h(y) = \bar{\theta} + \frac{(\bar{\theta}_{max} - \bar{\theta}_{min}) \arctan(\delta y)}{\pi}. \quad (30)$$

While unusual, (30), depicted in Figure 3, constitutes a natural choice. The ability distribution determines the average ability in the two schools in the event of complete segregation by ability, when all the students whose ability is above the median are in one school. These are denoted by $\bar{\theta}_{max}$ and $\bar{\theta}_{min}$, and (30) implies that these can be reached only as the difference between the average qualifications in the two schools tends to ∞ . The only parameter of this adjustment function is therefore δ , which measures the speed with which parents respond to past difference in results. Figure 3 illustrates this function for two values of δ (the dashed line denotes the higher speed of adjustment).

l	\bar{q}_1	\bar{q}_2	\bar{q}_h	\bar{q}_l	x_1	x_2	e_1	e_2
Table 3a: $a = 0.1, d = 10, r = 0.3, k = 0.9, s = 10, g = 10$								
1	0.0585	0.0585	0.5000	0.5000	0.0536	0.0536	0.0953	0.0953
2	0.0883	0.0883	0.5000	0.5000	0.1080	0.1080	0.1000	0.1000
3	0.1759	0.0731	0.6272	0.3728	0.1975	0.1288	0.1043	0.1043
4	0.2443	0.0789	0.6635	0.3365	0.2779	0.1599	0.1086	0.1076
5	0.3102	0.0877	0.6828	0.3172	0.3577	0.1926	0.1122	0.1107
6	0.3751	0.0980	0.6949	0.3051	0.4374	0.2261	0.1151	0.1137
7	0.4392	0.1091	0.7032	0.2968	0.5168	0.2604	0.1171	0.1165
Table 3b: $a = -0.6, d = 10, r = 0.9, k = 0.9, s = 5, g = 3$								
1.5	0.1514	0.1514	0.5000	0.5000	0.1871	0.1871	0.1930	0.1930
1.55	0.1555	0.1550	0.5008	0.4992	0.1935	0.1932	0.1927	0.1933
1.6	0.1709	0.1476	0.5365	0.4635	0.2059	0.1931	0.1786	0.2069
1.65	0.1798	0.1467	0.5510	0.4490	0.2149	0.1965	0.1726	0.2124
1.7	0.1876	0.1468	0.5616	0.4384	0.2233	0.2004	0.1681	0.2166
1.75	0.1948	0.1476	0.5702	0.4298	0.2314	0.2046	0.1642	0.2199
2	0.2267	0.1549	0.5991	0.4009	0.2697	0.2276	0.1500	0.2314
3	0.3299	0.2000	0.6456	0.3544	0.4105	0.3272	0.1146	0.2516
4	0.4133	0.2505	0.6623	0.3377	0.5394	0.4293	0.0868	0.2585
5	0.4821	0.3010	0.6697	0.3303	0.6581	0.5306	0.0615	0.2593
6	0.5390	0.3497	0.6726	0.3274	0.7678	0.6299	0.0382	0.2559
7	0.5864	0.3956	0.6732	0.3268	0.8694	0.7263	0.0169	0.2496

Table 3:
Effects of changes in the power of incentives on the equilibrium of the dynamic game.

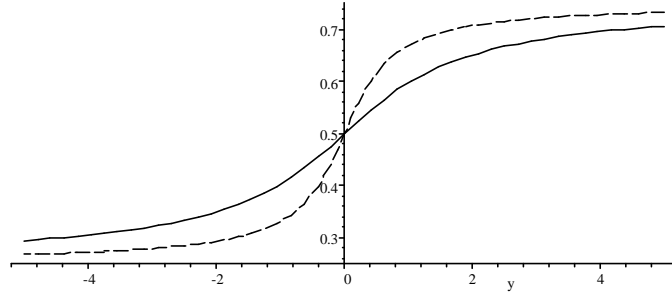


Figure 3: A school's average ability as a function of the difference in investment

Table 3 illustrates the stable equilibrium for two sets of simulations, where λ increases in each set for the given values of the other parameters. These have been obtained data by starting from an initial average ability pair $(\frac{1}{2} + \varepsilon, \frac{1}{2} - \varepsilon)$, and running the dynamical system until the difference between the ability in each school in successive time periods was below a preset threshold.

A pattern common to all the simulations we have ran is that, as λ increases, the symmetric equilibrium ceases to be stable: *increasing the power of incentives increases segregation*. This is due to the fact that, at an equilib-

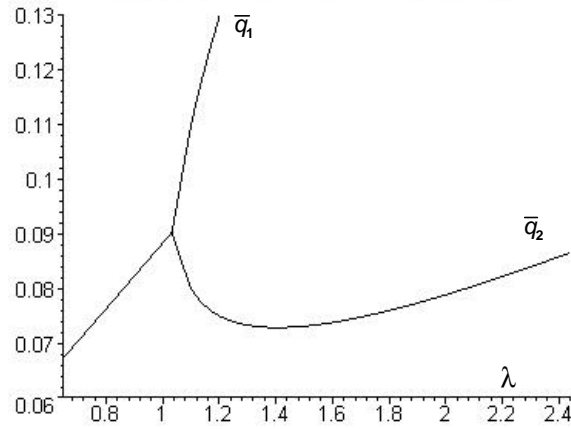


Figure 4: Value of the average qualification in the two schools

rium, an increase in λ makes it worthwhile for the better school to increase effort; this increases its average qualification, and so its attractiveness to students. Given our model specification, the effects is stronger for the better school, although it is conceivable that the opposite might be the case. The effects on effort are ambiguous. In Table 3a an increase in λ increases the efforts of both schools and the effort of the students in both schools. Notice also that students and teachers work harder in the better school. In the lower part of the Table, the effect on effort is ambiguous: as λ increases, teachers work harder in the better school while students work less hard. The same is true in the lower ability school, when the incentives are sufficiently powerful.

Note also that, when λ is such that the schools are not very different in intake, an increase in λ has opposite effects on the qualification of the two schools: the “better” school responds to an increase in λ with an improvement in the qualification, the other school with a reduction (in Table 3b this happens for a smaller range of values for λ). This pattern is illustrated in Figure 4 which shows the average qualification in the two schools as a function of λ , and is caused by the reduction in the average ability of the intake, accompanied by the possible reduction in the school’s effort illustrated in the seventh column in Table 3b.

In Table 4 we consider the effects of changes in δ , the effectiveness of competition between schools. We find that when competition is weak the symmetric equilibrium $(\frac{1}{2}, \frac{1}{2})$ is stable. As δ increases, small initial differ-

d	\bar{q}_1	\bar{q}_2	\bar{q}_1	\bar{q}_2	x_1	x_2	e_1	e_2
Table 4a: $a = 0.1, l = 5, r = 0.3, k = 0.5, s = 10, g = 5$								
3	0.2490	0.2490	0.5000	0.5000	0.1824	0.1824	0.2296	0.2296
5	0.3577	0.1583	0.6247	0.3753	0.2134	0.1505	0.2287	0.2269
7	0.4058	0.1274	0.6745	0.3255	0.2254	0.1376	0.2272	0.2249
9	0.4267	0.1154	0.6954	0.3046	0.2304	0.1321	0.2263	0.2240
Table 4b: $a = 1, l = 5, r = 0.3, k = 0.5, s = 10, g = 5$								
3	0.2857	0.2857	0.5000	0.5000	0.2031	0.2031	0.3125	0.3125
5	0.4705	0.1485	0.6615	0.3385	0.2563	0.1515	0.3637	0.2677
7	0.5130	0.1268	0.6936	0.3064	0.2670	0.1416	0.3744	0.2598
9	0.5332	0.1174	0.7084	0.2916	0.2720	0.1370	0.3794	0.2563
Table 4c: $a = 0.1, l = 5, r = 0.9, k = 0.5, s = 10, g = 5$								
3	0.2235	0.2235	0.5000	0.5000	0.1674	0.1674	0.1696	0.1696
5	0.2922	0.1632	0.5912	0.4088	0.1876	0.1468	0.1592	0.1784
7	0.3497	0.1232	0.6604	0.3396	0.2026	0.1309	0.1499	0.1842
9	0.3723	0.1096	0.6863	0.3137	0.2081	0.1250	0.1462	0.1862
Table 4d: $a = 0.1, l = 5, r = 0.9, k = 0.5, s = 10, g = 0.7$								
5	0.7204	0.7204	0.5000	0.5000	0.4001	0.4001	1.1006	1.1006
7	0.7972	0.6335	0.6358	0.3642	0.3890	0.4094	0.9200	1.2735
9	0.8158	0.6088	0.6716	0.3284	0.3858	0.4114	0.8715	1.3174
11	0.8251	0.5960	0.6899	0.3101	0.3841	0.4124	0.8466	1.3395
Table 4e: $a = 0.1, l = 5, r = 0.6, k = 0.3, s = 10, g = 0.8$								
2	0.6992	0.6992	0.5000	0.5000	0.2473	0.2473	1.1484	1.1484
4	0.8803	0.5016	0.6571	0.3429	0.2472	0.2410	0.9910	1.2635
6	0.9216	0.4518	0.6957	0.3043	0.2464	0.2382	0.9472	1.2840
8	0.9380	0.4314	0.7115	0.2885	0.2460	0.2370	0.9288	1.2915

Table 4:
Effects of changes in the effectiveness on the equilibrium of the dynamic game.

ences are amplified, and the better school becomes better, increasing both average ability of the intake and average qualification. The other school becomes worse both in terms of intake and of qualifications. Competition generates segregation.²³

What seems to drive the students' attainment in the two schools, however, is the change in intake, as the wide variety of patterns which emerges with regard to effort would suggest. This is shown in the last four columns of Table 4. Of particular interest is the set of parameters given in Table 4d. Here results improve in school 1 and worsen in school 2 as competition becomes more effective, despite that fact that students and teachers work more in the weaker school and less in the better school, and indeed both the students and the school work harder in the weaker school.

²³This conclusion is in line with the empirical analysis of Bradley and Taylor (2002), who find that, in the UK, where recent reforms have increased parental choice with regard to the school attended by their children, have also led to higher segregation of pupils.

5 Concluding remarks.

The paper analyses the effects of incentives in the education sector and competition between education institutions. A school's results depend on the school's investment and the school's teaching effort, both of which are directly affected by the incentives provided to schools, and by the students' effort in learning.

The main messages of the paper are simply put: *incentives may back-fire and competition may have perverse effects*. This is due to the strategic interaction among the participants in the education process. For example, students may reduce their effort when teachers increase theirs. This may dampen the effect of increases in the power of incentives on the results. When schools interact with one another, their attempts to attract the better students further blurs the relationship between agents, to the point, as we show with robust and plausible functional forms and parameter combinations, that increases in the power of incentives and the effectiveness of competition may reduce the students' attainment. To the extent that incentives are costly, our analysis may be interpreted as providing a theoretical underpinning for the ambiguous relationship between resources and result which some literature has identified (e.g., Hanushek 1986). At the very least, our paper illustrates the importance of further theoretical research to clarify the interaction between schools and students.

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